



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS APRIL-JANUARY 2025

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 410

COURSE TITLE: COMPLEX ANALYSIS

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

a) Determine the value of $\ln(1 - i)$ and state its principle value. (5 marks)

b) Expand $\frac{1}{(z+3)(z+1)}$ in a Laurent series valid for $0 < |z + 1| < 2$. (4 marks)

c) Use Cauchy's integral formula to evaluate,

$$\oint \frac{\cos z}{z(z^2+8)} dz. \quad (5 \text{ marks}).$$

d) Simplify $\frac{(\cos \theta - i \sin \theta)(\cos 7\phi + i \sin 7\phi)}{(\cos 41\phi + i \sin 41\phi)(\cos 5\theta - i \sin 5\theta)}$ (3 marks)

e) Differentiate $f(z) = \frac{2}{z^2}$ from the first principles. (4 marks)

f) Find the value of $(3 - i\sqrt{2})^{40}$. (3 marks)

g) Find the transformation that maps $z = 2, i, -2$ in the z -plane into $w = 1, i, -1$ in the w -plane. (6 marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

a) Determine the values of the constants m and n which make the function,

$$f(z) = \begin{cases} z + 1 & , z < 1 \\ mz + n & , 1 \leq z < 2 \\ 3z & , z \geq 2 \end{cases}$$

Continuous for all values z . (5 marks).

b) Explain why you cannot apply Cauchy – Goursat theorem on

$$\oint \frac{dz}{z^3},$$

Where C: $|z| = 1\frac{1}{2}$. (2 marks)

c) Expand $f(z) = \frac{z}{(1+z)(3+z)}$ into a Laurent series valid for $1 < |z| < 3$.
(7 marks)

d) Evaluate the integral $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the parabola,
 $x = 2t, y = t^2 + 3$ (6 marks)

QUESTION THREE (20 MARKS)

- a) Use the Cauchy's Residue Theorem to evaluate the integral $\oint \frac{1-2z}{z(z-1)(z-3)} dz$,
along $|z| = 2$. (5 marks)
- b) Evaluate the poles and corresponding residues for the function $f(z) = \frac{e^{iz}}{z^2(z^2 + 2z + z)}$. (5 marks)
- c) Derive the Laplace equations. (5 marks)
- d) Evaluate $Z^4 = -\sqrt{3} - i$. (5 marks)

QUESTION FOUR (20 MARKS)

- a) Construct an analytic function when $\text{real}(z) = x^3 - 3xy^2 + 3x^2 - 3y^2$.
(6 marks)
- b) Expand $f(z) = \frac{1}{z-1}$ as a Taylor series about $z=0$. (6 marks)
- c) State without proof the Cauchy – Riemann equations. (2 marks)
- d) Use the equations stated in 4 (a) above to determine whether,
 $f(z) = z^2 + 5iz + 3 - i$ is analytic. (6 marks)

QUESTION FIVE (20 MARKS)

- a) Show that $\sin hz = \sin hx \cos y + i \cos hx \sin y$ (4 marks)
- b) Evaluate $\oint \frac{dz}{z^2 - iz + 6}$ over $C : |z - i| = 1$. (7 marks)
- c) Use De Moivre's theorem to show that $\frac{\sin 4\theta}{\sin \theta} = 8\cos^3 \theta - 4\cos \theta$. (5 marks)
- d) Is the function $f(x, y) = xe^x \cos y - ye^x \sin y$ harmonic?. (4 marks)