



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY - APRIL 2025 SEMESTER

BSc. COMPUTER SCIENCE

COURSE CODE: BSCS 104

COURSE TITLE: DISCRETE STRUCTURES II

DATE: APRIL 2025

TIME: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

- a) Define the terms ‘witness’ and ‘universe of discourse’ as used in quantification. (4 marks)
- b) State the truth value of $\exists x P(x)$ in the domain of integers given that $P(x) = x^2 \leq 3$ (3 marks)
- c) Using contrapositive, prove that if $3n + 2$ is even, then n is even. (4 marks)
- d) Prove by mathematical induction that $2 + 4 + 6 + \dots + 2n = n(n+1)$. (4 marks)
- e) Using direct proof show that the product of two odd numbers is odd (4 marks)
- f) When a die was thrown, the score was an odd number. What is the probability that it was a prime number (4 marks)
- g) Construct a truth table for $\sim p \wedge (p \wedge q)$. (5 marks)
- h) Differentiate between a graph and a tree (2 marks)

SECTION B: ANSWER ANY TWO QUESTIONS

Question Two

- a) Proof by contraposition that if $3n+2$ is even, then n is even (6 marks)
- b) Let $P(x)$ be the statement “ $x \leq 1 \leq x$ ”. What is the truth value of the quantification $\exists x P(x)$ where the domain is the set of real numbers (4 marks)
- c) Define the term spanning tree as used in graphs. (2 marks)
 - i) Let G be the graph with the following adjacency list
 - a: b
 - b: a, c, e
 - c: b, d, e
 - d: c
 - e: b, c
 - ii) Draw graph G (4 marks)
 - iii) Draw at least 2 spanning trees of G (4 marks)

Question Three

- a) State the Bayes’ Theorem (2 marks)
- b) A jar contains 3 black marbles and 4 white marbles. A marble is picked at random without replacement. What is the probability of getting a black marble the second time given that the first one was also black (4 marks)
- c) Solve the recurrence relation $an \leq 10an \leq 1 \leq 25an \leq 2$, with initial conditions $a_0 \leq 3$ and $a_1 \leq 17$ (6 marks)
- d) State and prove the handshake Lemma (4 marks)
- e) Given the following propositional function, determine its true/false value when the variables are set as follows;

$C(x, y) = \text{“}x \text{ is the capital of } y\text{”}$

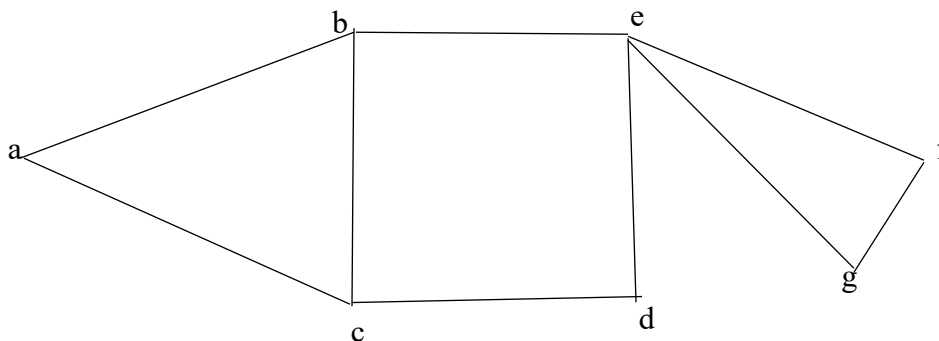
- a) C(Ottawa, Canada) (2 marks)
 b) C(Kampala, Rwanda) (2 marks)

Question Four

- a) Define
- Mutually exclusive events (1 mark)
 - Independent events (1 mark)
- b) A number is chosen at random from 1,2,3,...,30. Find the probability that
- It is divisible by 4 (1 mark)
 - It is divisible by both 3 and 4 (2 marks)
 - It is divisible by 4 given that it's divisible by 3 (2 marks)
- c) Prove that “the difference between the squares of any two consecutive even numbers is twice the sum of the two numbers” using direct proof (5 marks)
- d) Events T and R are such that the probability of $T' = \frac{3}{5}$. $P(R|T') = \frac{1}{3}$ while $P(R|T) = \frac{1}{4}$. Evaluate;
- $P(R)$ (4 marks)
 - $P(T'|R)$ (4 marks)

Question Five

- a) Define the following terms as used in graphs.
- Circuit (1 mark)
 - Path (1 mark)
- b) Given the following graph



Using the graph, find

- A walk from b to d (2 marks)
- A trail from b to d (2 marks)
- A circuit from b to b (2 marks)

- iv.** A cycle from e to e (2 marks)
- a) Consider $A = \begin{pmatrix} 4 & -1 & 1 \\ -3 & 4 & 5 \\ 2 & 3 & 2 \end{pmatrix}$; Evaluate,
- i.** $|A|$. (2 marks)
- ii.** M_{12} , a_{23} and A_{23} . (3 marks)
- iii.** A^{-1} (5 marks)



GRETSA UNIVERSITY – THIKA

**UNIVERSITY EXAMINATIONS
JANUARY - APRIL 2025 SEMESTER**

BACHELOR OF EDUCATION ARTS

**COURSE CODE: BSCS 209/109
COURSE TITLE: ANALYTICAL GEOMETRY**

DATE: APRIL 2025

TIME: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
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SECTION A: COMPULSORY

Question One

- a) State and sketch the vertex and focus of the parabola having the equation.

$$(y - 3)^2 = 8(x - 5) \quad [5 \text{ Marks}]$$

- b) Evaluate the angle between $y - 2x = 2$ and $y + x = 4$ [6 Marks]

- c) Given the circle with equation $x^2 + y^2 - 6x + 2y + 8 = 0$. Rewrite the equation in the standard form and give the coordinates of the centre and the radius. [5 Marks]

- d) A line passes through $(-3, 9)$ and $(4, 4)$. Another line passes through $(9, -1)$ and $(4, -8)$. Are the two lines parallel or perpendicular? [4 Marks]

- e) Find the equation of a straight line passing through the point $(4, -5)$ and which is perpendicular to the line $3x + 4y + 5 = 0$ [5 Marks]

- f) Show that

$$\frac{(1 - \cos \theta)(1 + \cos \theta)}{(1 - \sin \theta)(1 + \sin \theta)} = \tan^2 \theta$$

SECTION B: ANSWER ANY TWO QUESTIONS

Question Two

- a) Given the point A (3, -2) and B (2, 4). Find the equation of the perpendicular bisector of AB. [5 Marks]
- b) Find the length of a tangent drawn from the point (2,8) to the circle $x^2 + 4x + y^2 + 10y + 20 = 0$. [5 Marks]
- c) Find all the solutions of the equation $12\cos x + 5\sin x + 3 = 0$ in the range $0 \leq x \leq 2\pi$, give your answers in radians. [7 Marks]
- d) Convert (-1, -1) to polar form. [3 Marks]

Question Three

- a) The equation of a circle is given by $x^2 + y^2 = 10$. Prove that $y = 3x + 10$ is the tangent to the circle. [6 Marks]
- b) Given that the point $A(x, 3)$ is 5 units distant from another point $B(-1, 0)$. Find the value of x . [5 Marks]
- c) Find the radius of a circle which passes through the vertices of triangle ABC with sides $AB=3\text{cm}$, $AC=2.5\text{cm}$ and $\angle ABC = 49^\circ$. [5 Marks]
- d) If $\tan \theta = \frac{1}{a}$, show that $\frac{\cos \theta + \sin^2 \theta + \cos^3 \theta}{\sin \theta} = \frac{1}{a}$ [4 Marks]

Question Four

- a) The equation of a circle is given by $x^2 + y^2 = 10$. Prove that $y = 3x + 10$ is the tangent to the circle. [5 Marks]
- b) If A(-2, 1), B(2,3) and C(-2, -4) are three distinct points, find the angle between the straight line AB and BC. [5 Marks]
- c) Given the focus (-6, 0) and the vertex at the origin. Graph the parabola and write the equation of the parabola. [5 Marks]
- d) Graph the hyperbola given by the equation below [5 Marks]

$$\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$

Question Five

- a) Graph the ellipse given by the equation below

$$25x^2 + y^2 = 25$$

$$10 \leq x \leq 10$$

[5 Marks]

- b) Convert $(16, \frac{\pi}{2})$ to rectangular form.

[5 Marks]

- c) Determine the length of the tangent drawn from $P(5,6)$ to the point of contact with

the circle $x^2 + 6x + y^2 + 10y + 14 = 0$.

[5 Marks]

- d) Given that $2\sin^2 \theta = \sin \theta$, find θ in the range $0^\circ \leq \theta < 360^\circ$.



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 SEMESTER

BACHELOR OF INFORMATION TECHNOLOGY

COURSE CODE: BSCS 203

COURSE TITLE: DIFFERENTIAL AND INTEGRAL CALCULUS

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
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SECTION A: COMPULSORY

Question One

- a) Evaluate $\lim_{x \rightarrow 1} \frac{1-\sqrt{x}}{1-x}$ (4 Marks)
- b) From first principles, show that; If $y = k$ (constant) then $\frac{dy}{dx} = 0$ (2 Marks)
- c) Determine the range and domain of the function $f(x) = \sqrt{4-7x}$ (3 marks)
- d) Given $f(x) = 3x - 2$ find $(f \circ f^{-1})(x)$ (5 Marks)
- e) Given $2y^2 - 5x^4 - 2 - 7y^3 = 0$, determine $\frac{dy}{dx}$ (4 Marks)
- f) Consider the function

$$f(x) = \begin{cases} 7x - 2, & x \geq 2 \\ 3x + 5, & x < 2 \end{cases}$$

Determine the one sided limits

- i. $\lim_{x \rightarrow 2^+} f(x)$ (2 Marks)
- ii. $\lim_{x \rightarrow 2^-} f(x)$ (2 Marks)
- g) Evaluate $\frac{dy}{dx}$ in the following:
- i. $y = \left(x + \frac{4}{x}\right)(x^2 - 4)$ (4 Marks)
- ii. $y = \frac{3x^2 + 5x + 1}{3 - x^2}$ (4 Marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO

- a) Define continuity of a function at a point a (3 Marks)
- b) Find the gradient of the curve $y = 3x^4 - 2x^2 + 5x - 2$ at the point $(0, 2)$ (3 Marks)
- c) Evaluate $\lim_{x \rightarrow 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$ (3 Marks)
- d) Compute $\frac{d^2y}{dx^2}$ for the parametric equations $x = \cos 2t, y = \sin 2t$ (5 Marks)
- e) Consider the function $f(x) = x^4 - 2x^2$. Determine using the second derivative test:
- i. Maximum point (3 Marks)
- ii. Minimum point (3 Marks)

QUESTION THREE

- a) Given $f(x) = 3x^2 - x + 10$ and $g(x) = 1 - 20x$ find $(f \circ g)(x)$ and hence $(f \circ g)(5)$ (5 Marks)
- b) Evaluate
- i. $\lim_{x \rightarrow 1} \frac{1-\sqrt{x}}{1-x}$ (4 Marks)
- ii. $\lim_{x \rightarrow 1} \frac{x^4 + 3x^3 - 13x^2 - 27x + 36}{x^2 + 3x - 4}$ (5 Marks)

- iii. $\lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2}$ (3 Marks)
- iv. $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3}$ (3 Marks)

QUESTION FOUR

- a) Determine the inverse of the function $f(x) = \sqrt{4-x}$ (3 Marks)
- b) Given $y = (12x^3 - 4x^2 - x + 1)^4$, find $\frac{dy}{dx}$ (2 Marks)
- c) Given two functions $f(x) = 3x - 2$, $g(x) = 8 - 9x$, determine $(g \circ f)(x)$ (3 Marks)
- d) A particle is moving in a straight line and its distance s metres from a fixed point in the line after t seconds is given by the equation $s = 12t - 15t^2 + 4t^3$. Determine:
- The velocity of the particle after 3 seconds (2 Marks)
 - Acceleration of the particle after 3 seconds (2 Marks)
- e) Given the function $y = 2e^{2t}$, show that $y'' - 4y = 0$ (3 Marks)
- f) Using first principles find $f'(x)$ given $f(x) = \sin x$ (5 Marks)

QUESTION FIVE

- a) Determine $\frac{dy}{dx}$ given
- $5y^2 + \sin y = x^2$ (3 Marks)
 - $y = 3^{\cos x} x^x$ (5 Marks)
- b) Consider the Folium of Descartes equation $x^3 + y^3 - 6xy = 0$
- Determine its tangent slope at the point $(\frac{4}{3}, \frac{8}{3})$ (3 Marks)
 - Calculate its normal line equation at the point $(\frac{4}{3}, \frac{8}{3})$ (3 Marks)
- c) The parametric equations of a curve are as follows: $x=e^t$, $y=\sin t$. Find
- $\frac{dy}{dx}$ (2 Marks)
 - $\frac{d^2y}{dx^2}$ (2 Marks)
 - Hence show that $x^2 \left(\frac{d^2y}{dx^2} + \frac{dy}{dx} \right) + xy = 0$ (2 Marks)



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 SEMESTER

BACHELOR OF EDUCATION(ARTS)

COURSE CODE: BSCS 210

COURSE TITLE: LINEAR ALGEBRA 1

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
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SECTION A: COMPULSORY

Question One

a) Consider $A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 4 & 5 \\ 2 & 3 & 6 \end{pmatrix}$; Evaluate,

i. $|A^2|$. (3 marks)

ii. A^{-1} (5 marks)

b) Determine whether $\{(1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5)\}$ is a basis of $\mathbb{R}^4$. (3 marks)

c) Find the distance between the parallel planes $2x - y + z = 1$ and $2x - y + z = -1$. (3mks)

d) Given that $A = \begin{pmatrix} 2 & 4 \\ -23 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 4 \\ 7 & -15 \end{pmatrix}$, find $4A + 2B^T$. (3 marks)

e) Express $P = t^2 + 4t - 3$ as a linear combination of the polynomials, $p_1 = t^2 - 2t + 5$, $p_2 = 2t^2 - 3t$, $p_3 = t + 3$. (5 marks)

f) Determine whether or not the following vectors are orthogonal.
 $\vec{a} = (\sin\theta, -\cos\theta)$, $\vec{b} = (\cos\theta, \sin\theta)$. (3 marks)

g) Solve the following system using Gauss-Jordan elimination method. (5 marks)

$$x + y + z = 9$$

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0$$

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Are the following vectors linearly independent? (3 marks)
- i. $(1, -4, -3, 1), (1, -1, -1, 2), (3, -8, -2, 6)$. (3 marks)
- ii. $\vec{a} = t^2 - 4t + 2, \vec{b} = t^2 + 2t + 4, \vec{c} = 2t^2 - t - 3$. (3 marks)
- b) If A (-2, 1), B (2, 3) and C (-2, -4) are three points, find the angle between lines AB and BC. (5 marks)
- c) Define a spanning set, hence/ otherwise, show that the set $\{(1, 1, 0), (-1, 1, 0), (1, 2, 1)\}$ spans $\mathbb{R}^3$. (5 marks)
- d) Write $(1, -3, 2)$ as a linear combination of $(2, -1, 1)$ and $(1, -8, 5)$. (4 marks)

QUESTION THREE (20 MARKS)

- a) Solve the following system by Gaussian elimination method. (5 marks)

$$2x + y + 2z = 10$$

$$x + 2y + z = 8$$

$$3x + y - z = 2$$

- b) Solve the following system using the Cramer's Rule method. (5 marks)

$$x_1 + 3x_2 + 2x_3 = 3$$

$$2x_1 + 4x_2 + 2x_3 = 8$$

$$x_1 + 2x_2 - x_3 = 10$$

- c) Find the parametric equations and symmetric equations of a line passing through the points $(2, -3, 4)$ and $(3, 5, -6)$. (5 marks)
- d) Find the reduced row echelon form of the following matrix. (5 marks)

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 5 \end{pmatrix}$$

QUESTION FOUR (20 MARKS)

- a) Let W be the subspace of $\mathbb{R}^4$ spanned by vectors $(1, 2, 0, -2), (2, 6, -3, -3), (3, 10, -6, -5)$. Find a basis and dimension of W . (5 marks)
- b) Find the parametric and the symmetric equations of the line which passes through the point $(4, -1, 9)$ and is perpendicular to the plane $3x - 2y + z = 18$. (3 marks)
- c) Determine whether the following vectors are linearly dependent or linearly independent. $(1, -3, 2), (2, -4, -1), (1, -5, 7)$. (3 marks)
- b. Show that the line $y = 2x + 3$ is a subspace of $\mathbb{R}^2$ (4 marks)
- c. Find the inverse of the below matrix by row echelon reduction form. (5 marks)

$$\begin{pmatrix} 2 & 1 & -1 \\ 5 & 2 & -3 \\ 0 & 2 & 1 \end{pmatrix}$$

QUESTION FIVE (20 MARKS)

- a) Let $\vec{a} = (1, -3, 2)$ and $\vec{b} = (2, -1, 1)$. Find;
- i. A scalar β such that $\vec{a}$ is orthogonal to $\vec{a} - \beta\vec{b}$. (3 marks)
 - ii. A unit vector that is orthogonal to both $\vec{a}$ and $\vec{b}$. (3 marks)
- b) Determine whether or not the following are subspaces of $\mathbb{R}^3$.
- i. $U = \{(x, y, z) \in \mathbb{R}^3 \mid x = 2y\}$. (4marks)

ii. $V = \{(a, b, c) \in \mathbb{R}^3 \mid ab = 0\}.$

(3marks)

c) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$

Find the inverse of the matrix A; hence use it to solve the following system of linear equations using the inverse method.

$$x + y + 2z = 0$$

$$x - y + z = 4$$

$$2x + y + z = 2$$

(7 marks)



GRE TSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 EMESTER

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 311

COURSE TITLE: ORDINARY DIFFERNTIAL EQUATIONS

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
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SECTION A: COMPULSORY

Question One

- a. State the order and degree of the D.E $\sqrt{y'''} + 2y' = 3x$ (3 Marks)
- b. Show that $y = c_1 \sin 2x + c_2 \cos 2x$ is a general solution to $y'' + 4y = 0$. Hence determine a particular solution given that $y(0) = 0$ and $y'(0) = 1$. (6 Marks)
- c. A certain radioactive material is known to decay at a rate proportional to the amount present. If initially 500mg of the material is present and after 3 years 20% of the original mass has decayed, find an expression for the mass at any time. (5 Marks)
- d. Using the method of separation of variables, solve $\frac{dy}{dx} = \frac{xy^2+x}{yx^2+y}$ (5 Marks)
- e. Show that $\frac{1}{x^2y^2}$ is an integrating factor to the equation $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ and solve. (6 Marks)
- f. Solve the Bernoulli equation $\frac{dy}{dx} = xy^4 - y$ (5 Marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 Marks)

- a. Solve the homogeneous equation $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$ (7 Marks)
- b. Test whether the equation $(x + y)^2dx - (y^2 - 2xy - x^2)dy = 0$ is exact and solve (8 Marks)
- c. For what values of k is $y = e^{kx}$ a solution to the differential equation $y'' - 3y' - 4y = 0$. (5 marks)

QUESTION THREE [20 Marks]

- a. Find the general solution to the differential equation $y'' + 4y' + 13y = 0$ (4 Marks)
- b. Solve $\frac{dy}{dx} = \frac{x-2y+5}{2x+y-1}$ (8 Marks)
- c. Use the method of undetermined coefficients to solve $(D^2 + 6D + 13)y = 13x^2 - x + 22$ (8 Marks)

QUESTION FOUR [20 Marks]

- a. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = cx$ (6 Marks)
- b. Solve the linear equation $\frac{dy}{dx} + \frac{y}{x} = e^x$ (4 Marks)
- c. Solve $y'' - y' - 2y = 4x^2 - 12x + 41$ by the method of variation of parameters. (10 Marks)

QUESTION FIVE [20 Marks]

- a. Solve $y'' - 6y' + 25y = 0, y(0) = -3, y'(0) = -1$ (6 Marks)
- b. Find an integrating factor of the equation $6xydx + (4y + 9x^2)dy = 0$ (4 Marks)
- c. Find the differential equation associated with $y = A\cos x + B\sin x$. (4 marks)
- d. Air at a temperature 200K is passed over a substance at 300K. The temperature of the substance cools down to 260K in 30 minutes. Assuming that the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air, find after what time the temperature of the substance would be 240K. (6 Marks)

END



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 SEMESTER

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 312

COURSE TITLE: LINEAR ALGEBRA 2

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
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SECTION A: COMPULSORY

Question One

- a) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a mapping defined by $T(x, y) = (x + y, x)$. Is T linear? (5 marks)
- b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 5 & 2 \\ 9 & 2 \end{pmatrix}$ (9 marks)
- c) Define the linear operator T on $\mathbb{R}^2$ by $T(x, y) = (2x + y, 4x - y)$. Compute the matrix representation of T relative to the basis $\{g_1 = (1, 1), g_2 = (-1, 0)\}$ (4 marks)
- d) Let H be the set of all vectors of the form $\begin{bmatrix} c - a \\ a - 6b \\ 2a + b \end{bmatrix}$. Show that H is a subspace (3 marks)
- e) Define an operator $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(x, y, z) = (2x, -y, z)$. Determine whether T is linear. (5 Marks)
- f) Determine whether vectors $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$ are linearly independent if $\vec{V}_1 = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \vec{V}_2 = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$ and $\vec{V}_3 = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$ (4 marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Let $\vec{v}_1, \vec{v}_2 \in V$. Show that $H = \text{span}\{\vec{v}_1, \vec{v}_2\}$ is a subspace of V (6 marks)
- b) Determine whether the matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ is a zero of the polynomial $P(x) = x^3 + 2x^2 - 4x + 3$ (8 marks)
- c) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $f(t) = 2t - 3$. Find $f(A)$ (3 marks)
- d) Determine which properties of a vector space are not satisfied by $\mathbb{N}$ - the set of natural numbers. (3 marks)

QUESTION THREE (20 MARKS)

- a) Define a linear map T on $\mathbb{R}^3$ by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find:
- A basis and the dimension of the image U of T
 - A basis and the dimension of the kernel W of T
- (10 Marks)

b) Find the characteristic polynomial, eigenvalues and minimal polynomial of the matrix

$$A = \begin{pmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix} \quad (10 \text{ marks})$$

QUESTION FOUR (20 MARKS)

a) Let $T: M_2\mathbb{R} \rightarrow \mathbb{R}_2[x]$ be defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a+2b+3c+4d)x^2 + (2a-b+c)x + (3a-2b+c+2d)$. Find the matrix representation $[T]_E^F$ relative to the bases $E = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ and $F = \{x^2, x, 1\}$ for $M_2\mathbb{R}$ and $\mathbb{R}_2[x]$. (5marks)

b) Let V be the vector space of functions with basis $S = \{\sin t, \cos t, e^{3t}\}$ and let $D: V \rightarrow V$ be the differential operator defined by $D(f(t)) = \frac{d}{dt}(f(t))$. (4 marks).

c) Consider the following bases of $\mathbb{R}^2$.

$$S = \{e_1 = (1, -2), e_2 = (3, -4)\} \text{ and } S^* = \{f_1 = (1, 3), f_2 = (3, 8)\}$$

- Find the co-ordinates of $V = (a, b)$ relative to the basis S . (2 marks)
- Find the change of basis matrix P from S to S^* . (2 marks)
- Find the change of basis matrix Q from S^* to S . (1 mark)
- Verify that $Q = P^{-1}$. (1 mark)
- Show that $P^{-1}[v] = [v]_{S^*}$ for any vector $V = (a, b)$ in $\mathbb{R}^2$. (1 mark)

d) Suppose that B is similar to A , i.e. $B = P^{-1}AP$. Prove that $B^n = P^{-1}A^nP$ and so B^n is similar to A^n . (2 marks)

QUESTION FIVE (20 MARKS)

a) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find:

- The determinant of A
- $\text{Adj}(A)$

iii. A^{-1}

(10 Marks)

b)

i. State Cayley – Hamilton theorem.

ii. Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 5 & 3 \\ 2 & 10 \end{pmatrix}$

iii. Verify Cayley – Hamilton theorem for A

(10 Marks)



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 SEMESTER

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 317

COURSE TITLE: LINEAR ANALYSIS

DATE: JANUARY 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

- a) i) Give the meaning of a semi-norm. (2 marks)
- ii) Show that $p(x) = \sup_{n \geq 1} |x_n|$ is a semi norm on C , the space of all convergent sequences. (4 marks)
- b) Let C be the space of all convergent sequences. Define $\|x\| = \lim_{n \rightarrow \infty} |x_n|$. Show that $\|x\|$ is not a norm on C . (5 marks)
- c) Show that $T: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $T(x) = 2x - 1$ is not a linear operator. (4 marks)
- d) i) What do you understand by the term Metric. (4 marks)
- ii) In the set of $X = \mathbb{R}$ of all real numbers define on $\mathbb{R} \times \mathbb{R}$ by $d(x, y) = |x - y| \forall x, y \in \mathbb{R}$. Show that d is a metric on $\mathbb{R}$. (5 marks)
- e) If p_n is a sequence of paranorms in a linear space X , then $p(x) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{p_n(x)}{1 + p_n(x)}$ is a paranorm on X . (6 marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Let E be an inner product space, then for arbitrary vectors $x, y \in E$, $|\langle x, y \rangle|^2 \leq \langle x, x \rangle \langle y, y \rangle$ with equality if and only if vectors x and y are linearly dependent. Prove. (6 marks)
- b) State without proof the Banach fixed point Theorem. (2 marks)
- c) State and prove the parallelogram law. (5 marks)

- d) i) Define a Hilbert space. (1 mark)
- ii) Show that the space L_p with $p \neq 2$ is not an inner product space and hence not a Hilbert. (6 marks)

QUESTION THREE (20 MARKS)

- a) Let T be a bounded linear operator. Show that the null space $N(T)$ or kernel of T is closed. (4 marks)
- b) Show that if $T: X \rightarrow X$ where X is a normed space is continuous at the origin, then T is uniformly continuous on X . (5 marks)
- c) i) Give the meaning of an inner product space (IPS). (4 marks)
- ii) Show that the linear space $\mathbb{R}^n$ (Euclidean space) with $\langle, \rangle$ defined for an arbitrary vectors $X = \{x_1, x_2, \dots, x_n\}$ and $y = \{y_1, y_2, \dots, y_n\}$ in $\mathbb{R}^n$ by $\langle, \rangle = \sum_{i=1}^n x_i y_i$ is an inner product space. (7 marks)

QUESTION FOUR (20 MARKS)

- a) Show that $f(x) = \int_a^b x(t) dt$ where $x(t) \in C[a, b]$ is a linear and bounded functional on $C[a, b]$. (7 marks)
- b) Let $T: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $T(x) = 7x$. Show that T is a linear operator. (3 marks)
- c) Define a complete metric space, hence show that the set $S = (0, 1]$ which is a subspace of $\mathbb{R}$ is not complete. (5 marks)

- d) Let $X=L_\infty$, the space of all bounded sequences. Define $p(x)=\sup_{n \geq 1} |x_n|$. Show that P is a paranorm. (5 marks)

QUESTION FIVE (20 MARKS)

- a) Let $T: D(T) \rightarrow Y$ be a linear operator where $D(T) \subset X$ and $R(T) \subset Y$. Prove that if T^{-1} exists, then it is a linear operator. (4 marks)
- b) Define a Banach space. (1 mark)
- c) Giving 2 examples give the meaning of a fixed point. (3 marks)
- d) i. Define a norm. (3 marks)
- ii. Show that $\|X\| = (\sum_{n=1}^{\infty} |x_n|^2)^{\frac{1}{2}}$ is a norm ($1 \leq n \leq \infty$). (5 marks)
- e) A norm on the linear space X defines a metric on X by $d(x, y) = \|x - y\|$. Proof that d is induced by the norm. (4 marks)



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS APRIL-JANUARY 2025

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 410

COURSE TITLE: COMPLEX ANALYSIS

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

a) Determine the value of $\ln(1 - i)$ and state its principle value. (5 marks)

b) Expand $\frac{1}{(z+3)(z+1)}$ in a Laurent series valid for $0 < |z + 1| < 2$. (4 marks)

c) Use Cauchy's integral formula to evaluate,

$$\oint \frac{\cos z}{z(z^2+8)} dz. \quad (5 \text{ marks}).$$

d) Simplify $\frac{(\cos \theta - i \sin \theta)(\cos 7\phi + i \sin 7\phi)}{(\cos 41\phi + i \sin 41\phi)(\cos 5\theta - i \sin 5\theta)}$ (3 marks)

e) Differentiate $f(z) = \frac{2}{z^2}$ from the first principles. (4 marks)

f) Find the value of $(3 - i\sqrt{2})^{40}$. (3 marks)

g) Find the transformation that maps $z = 2, i, -2$ in the z - plane into $w=1, i, -1$ in the w - plane. (6 marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

a) Determine the values of the constants m and n which make the function,

$$f(z) = \begin{cases} z + 1 & , z < 1 \\ mz + n & , 1 \leq z < 2 \\ 3z & , z \geq 2 \end{cases}$$

Continuous for all values z . (5 marks).

b) Explain why you cannot apply Cauchy – Goursat theorem on

$$\oint \frac{dz}{z^3},$$

Where C: $|z| = 1\frac{1}{2}$. (2 marks)

c) Expand $f(z) = \frac{z}{(1+z)(3+z)}$ into a Laurent series valid for $1 < |z| < 3$.
(7 marks)

d) Evaluate the integral $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the parabola,
 $x = 2t, y = t^2 + 3$ (6 marks)

QUESTION THREE (20 MARKS)

- a) Use the Cauchy's Residue Theorem to evaluate the integral $\oint \frac{1-2z}{z(z-1)(z-3)} dz$,
along $|z| = 2$. (5 marks)
- b) Evaluate the poles and corresponding residues for the function $f(z) = \frac{e^{iz}}{z^2(z^2 + 2z + z)}$. (5 marks)
- c) Derive the Laplace equations. (5 marks)
- d) Evaluate $Z^4 = -\sqrt{3} - i$. (5 marks)

QUESTION FOUR (20 MARKS)

- a) Construct an analytic function when $\text{real}(z) = x^3 - 3xy^2 + 3x^2 - 3y^2$.
(6 marks)
- b) Expand $f(z) = \frac{1}{z-1}$ as a Taylor series about $z=0$. (6 marks)
- c) State without proof the Cauchy – Riemann equations. (2 marks)
- d) Use the equations stated in 4 (a) above to determine whether,
 $f(z) = z^2 + 5iz + 3 - i$ is analytic. (6 marks)

QUESTION FIVE (20 MARKS)

- a) Show that $\sin hz = \sin hx \cos y + i \cos hx \sin y$ (4 marks)
- b) Evaluate $\oint \frac{dz}{z^2 - iz + 6}$ over $C : |z - i| = 1$. (7 marks)
- c) Use De Moivre's theorem to show that $\frac{\sin 4\theta}{\sin \theta} = 8\cos^3 \theta - 4\cos \theta$. (5 marks)
- d) Is the function $f(x, y) = xe^x \cos y - ye^x \sin y$ harmonic?. (4 marks)



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 411

COURSE TITLE: REAL ANALYSIS

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

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SECTION A: COMPULSORY

Question One

- a) (i) Let S be a subset of $\mathbb{R}$. Give the definition of an accumulation point of the set S in $\mathbb{R}$. (2 marks)
- (ii) Let $S = \{(-1)^n + \frac{1}{n} ; n \in \mathbb{N}\}$. Find the accumulation point of the set S in $\mathbb{R}$. (1 mark)
- (iii) When is a set S in $\mathbb{R}$ said to be closed ?. (1 mark)
- (iv) Let $u_n = [-1, \frac{-1}{n}]$, $n \in \mathbb{N}$ be a collection of closed sets. Using this collection, show clearly that an infinite union of closed sets in $\mathbb{R}$ is not necessarily closed. (3 marks)
- b) Use definition of limit of a sequence to prove that $\lim_{n \rightarrow \infty} \frac{2n^2+1}{n^2+1} = 2$. (4 marks)
- c) Find the closure of the set $A = \{(-1)^n \frac{n}{n+1}\}$ (3 marks)
- d) Test for absolute or conditional convergence of the series $\sum_1^\infty \frac{(-1)^{n-1}}{n^2+1}$. (4 marks)
- e) Prove that $\sqrt{3}$ is irrational (4 marks)
- f) The intersection of any finite collection of open sets is open in $\mathbb{R}$. Prove. (5 marks)
- g) Given that $A = \{2,3\}$ and $B = \{x,y,z\}$, find $A \times B$ and $A \times A$ (2 marks)

- h) State the completeness axiom of real numbers. (1 mark)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Show that $A = \{\frac{1}{2^n} : n \in \mathbb{N}\}$ is Cauchy (5 marks)
- b) Suppose that $\{x_n\}$ and $\{y_n\}$ are convergent sequences with $x_n \rightarrow x$ and $y_n \rightarrow y$ as $n \rightarrow \infty$. Show that $\lim_{n \rightarrow \infty} (x_n + y_n) = x + y$ (5 marks)
- c) Show that the sequence (x_n) in $\mathbb{R}$ has a unique limit. (5 marks)
- d) Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0, 1)$. (5 marks)

QUESTION THREE (20 MARKS)

- a) Show that the function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}; & x \neq 1; \\ -15 & x = 1; \end{cases}$

Is discontinuous at $x=1$. Hence/ otherwise, redefine $f(x)$ so that it is continuous at $x=1$. (6 marks)

- b) Show that finite sets have no limit points (4 marks)
- c) Is the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^5}}$ conditionally convergent? (hint; use alternating series and p- series test) (6 marks)
- d) Show that $\sum_{n=1}^{\infty} \frac{3}{10^n}$ converges. (4 marks)

QUESTION FOUR (20 MARKS)

- a) Use the algebraic properties of all real numbers $\mathbb{R}$ to show that $x \cdot 0 = 0$ for all $x \in \mathbb{R}$. (3 marks)
- b) Use Mathematical induction to prove that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 for all natural numbers n . (5 marks)
- c) Find the supremum and infimum (if they exist) of each of the following sets. Again, decide which sets have maximum and minimum elements respectively.
- $S = \{x \in \mathbb{R} ; x^2 < 5 \text{ and } x \text{ is a rational}\}$. (2 marks)
- $T = \{\frac{1}{n} + (-1)^2; n \text{ is a natural number}\}$. (2 marks)
- d) Suppose that r is a rational number and s is an irrational number such that $t = r + s$. Prove that t is irrational. (4 marks)
- e) Prove that every convergent sequence is bounded. (4 marks)

QUESTION FIVE (20 MARKS)

- a) Show that $w = \sqrt{n+1} - \sqrt{n-1}$ is irrational. (5 marks)
- b) Prove that every convergent sequence is Cauchy. (4 marks)
- c) Determine whether $f(x) = x^2 + 2x + 6$ is continuous at $x = 3$. (3 marks)
- d) Let $a_n = \begin{cases} \frac{n}{2^n}; & n \text{ is odd} \\ \frac{1}{2}, & n \text{ is even} \end{cases}$. Does the series $\sum a_n$ converge? (2 marks)
- e) Which of the following series converge, which one diverge
- i) $\sum \frac{n^2}{2^n}$. (3 marks)

ii) $\sum n! e^{-n}$

(3 marks)



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS

JANUARY-APRIL 2025

CERTIFICATE IN INFORMATION TECHNOLOGY

COURSE CODE: CFIT 014

COURSE TITLE: BASIC MATHEMATICS

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

- a) Find a number such that when it is divided by 3 and 2 added, the result is 17 (3 Marks)
- b) Using quadratic formulae, evaluate (5 Marks)
- $$9p^2 + 24p + 16 = 0$$
- c) Solve the equation $\log_5 x + \log_5(2x - 3) = 1$. (4 marks)
- d) Given the matrix $A = \begin{pmatrix} 2 & 3 \\ -4 & 7 \end{pmatrix}$ Find
- i. $\text{Det } A$ [2 Marks]
- ii. A^{-1} [2 Marks]
- e) Solve $x^2 + 4x - 21 = 0$ (4 Marks)
- f) Solve using elimination method (3 Marks)
- $$3x - 2y = 8$$
- $$x + y = 6$$
- g) For the series, $2 + 5 + 8 \dots$ find
- i. Common difference (1 mark)
- ii. The 10th term (3 Marks)
- iii. The sum of the first 100 terms (3 Marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & -1 & 6 \\ 1 & -1 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 1 \\ 3 & -1 & 4 \\ 1 & 0 & 4 \end{pmatrix}$ show that $A + B = B + A$ (4 Marks)

- b) A mixed hokey team containing 5 men and 6 women is to be chosen from 7 men and 9 women. In how many ways can this be done?. (4 marks)
- c) Using any valid formular, solve $2x^2 - 5x - 3 = 0$ (4 Marks)
- d) Rationalize the denominator $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{6}}$. (4 marks)
- e) Express $6\sqrt{5}$ in the form $\sqrt{p}$, where p is an integer. (4 marks)

QUESTION THREE (20 MARKS)

- a) Determine the first six terms of the AP whose first term is 18 and common difference is -5. (4 marks)
- b) Solve the simultaneous equations using substitution method (4 Marks)
- $$-2x + 3y = 3$$
- $$x + 3y = 4$$
- c) Expand $(1 - 3x)^{-2}$ as a series of ascending powers of x up to and including the term in x^3 . (5 marks)
- d) Solve for x in the equation $\frac{x+8}{5} = \frac{x-3}{-9}$. (4 marks)
- e) Expand $(2 + x^2)^3$ using binomial expansion (3 marks)

QUESTION FOUR (20 MARKS)

- a) Simplify and evaluate without the use of a calculator or tables the following expression, (5 marks)
- $$2\cos^2(45) \cos[44] + \tan^2(60)\sin(46) - 9\cos(60)\cos[44]$$
- b) Using the following set of numbers, identify all $\{-4, 0, \sqrt{-6}, -1, e, 1.825, \pi, \sqrt{2}, 81\}$
- (i) Natural numbers (1 mark)
- (ii) Integers (1 mark)
- (iii) Rational numbers (1 mark)
- (iv) Irrational numbers (1 mark)
- c) Using completing the square method, evaluate $x^2 + 4x - 12 = 0$ (5 Marks)
- d) Use binomial theorem to expand and simplify $(5 + 3x)^4$. Use the result to approximate $(5.09)^4$ (6 Marks)

QUESTION FIVE (20 MARKS)

a) Simplify $\frac{35a^7b^{12}c^{-13}}{7a^4b^6c^2}$. (5 marks)

b) The cost of three sheep and two goats is sh. 7200. If four sheep and a goat costs sh. 7600, find the cost of two goats and a sheep (5 marks)

c) Two years ago, a man was 7 times as old as his son. In 5 years time, he will be only 4 times as old as son. Find their respective ages. (5 marks)

d) Let $f(x) = 3x^3 - 5x^2 + 3x - 10$, find the remainder if $f(x)$ is divided by $x-1$ using long division (5 marks)



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 SEMESTER

DIPLOMA IN EDUCATION (ARTS)

COURSE CODE: DPMM 021

COURSE TITLE: LINEAR ALGEBRA

DATE: APRIL 2025

TIME: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

a) Consider $A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 4 & 5 \\ 2 & 3 & 6 \end{pmatrix}$; Evaluate,

i. $|A^2|$. (3 marks)

ii. A^{-1} (5 marks)

b) Determine whether $\{(1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5)\}$ is a basis of $\mathbb{R}^4$. (3 marks)

c) Find the distance between the parallel planes $2x - y + z = 1$ and $2x - y + z = -1$. (3mks)

d) Given that $A = \begin{pmatrix} 2 & 4 \\ -23 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 4 \\ 7 & -15 \end{pmatrix}$, find $4A + 2B^T$. (3 marks)

e) Express $P = t^2 + 4t - 3$ as a linear combination of the polynomials, $p_1 = t^2 - 2t + 5$, $p_2 = 2t^2 - 3t$, $p_3 = t + 3$. (5 marks)

f) Determine whether or not the following vectors are orthogonal. $\vec{a} = (\sin\theta, -\cos\theta)$, $\vec{b} = (\cos\theta, \sin\theta)$. (3 marks)

g) Solve the following system using Gauss-Jordan elimination method. (5 marks)

$$x + y + z = 9$$

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0$$

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Are the following vectors linearly independent? (3 marks)
- i. $(1, -4, -3, 1), (1, -1, -1, 2), (3, -8, -2, 6).$ (3 marks)
- ii. $\vec{a} = t^2 - 4t + 2, \vec{b} = t^2 + 2t + 4, \vec{c} = 2t^2 - t - 3.$ (3 marks)
- b) If A (-2, 1), B (2, 3) and C (-2, -4) are three points, find the angle between lines AB and BC. (5 marks)
- c) Define a spanning set, hence/ otherwise, show that the set $\{(1, 1, 0), (-1, 1, 0), (1, 2, 1)\}$ spans $\mathbb{R}^3$. (5 marks)
- d) Write $(1, -3, 2)$ as a linear combination of $(2, -1, 1)$ and $(1, -8, 5)$. (4 marks)

QUESTION THREE (20 MARKS)

- a) Solve the following system by Gaussian elimination method. (5 marks)

$$2x + y + 2z = 10$$

$$x + 2y + z = 8$$

$$3x + y - z = 2$$

- b) Solve the following system using the Cramer's Rule method. (5 marks)

$$x_1 + 3x_2 + 2x_3 = 3$$

$$2x_1 + 4x_2 + 2x_3 = 8$$

$$x_1 + 2x_2 - x_3 = 10$$

- c) Find the parametric equations and symmetric equations of a line passing through the points $(2, -3, 4)$ and $(3, 5, -6)$. (5 marks)
- d) Find the reduced row echelon form of the following matrix. (5 marks)

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 5 \end{pmatrix}$$

QUESTION FOUR (20 MARKS)

- a) Let W be the subspace of $\mathbb{R}^4$ spanned by vectors $(1, 2, 0, -2), (2, 6, -3, -3), (3, 10, -6, -5)$. Find a basis and dimension of W . (5 marks)
- b) Find the parametric and the symmetric equations of the line which passes through the point $(4, -1, 9)$ and is perpendicular to the plane $3x - 2y + z = 18$. (3 marks)
- c) Determine whether the following vectors are linearly dependent or linearly independent. $(1, -3, 2), (2, -4, -1), (1, -5, 7)$. (3 marks)
- b. Show that the line $y = 2x + 3$ is a subspace of $\mathbb{R}^2$ (4 marks)
- c. Find the inverse of the below matrix by row echelon reduction form. (5 marks)

$$\begin{pmatrix} 2 & 1 & -1 \\ 5 & 2 & -3 \\ 0 & 2 & 1 \end{pmatrix}$$

QUESTION FIVE (20 MARKS)

- a) Let $\vec{a} = (1, -3, 2)$ and $\vec{b} = (2, -1, 1)$. Find;
- A scalar β such that $\vec{a}$ is orthogonal to $\vec{a} - \beta\vec{b}$. (3 marks)
 - A unit vector that is orthogonal to both $\vec{a}$ and $\vec{b}$. (3 marks)
- b) Determine whether or not the following are subspaces of $\mathbb{R}^3$.
- $U = \{(x, y, z) \in \mathbb{R}^3 \mid x = 2y\}$. (4marks)

ii. $V = \{(a, b, c) \in \mathbb{R}^3 \mid ab = 0\}.$

(3marks)

c) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$

Find the inverse of the matrix A; hence use it to solve the following system of linear equations using the inverse method.

$$x + y + 2z = 0$$

$$x - y + z = 4$$

$$2x + y + z = 2$$

(7 marks)