



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 SEMESTER

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 312

COURSE TITLE: LINEAR ALGEBRA 2

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

- a) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a mapping defined by $T(x, y) = (x + y, x)$. Is T linear? (5 marks)
- b) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 5 & 2 \\ 9 & 2 \end{pmatrix}$ (9 marks)
- c) Define the linear operator T on $\mathbb{R}^2$ by $T(x, y) = (2x + y, 4x - y)$. Compute the matrix representation of T relative to the basis $\{g_1 = (1, 1), g_2 = (-1, 0)\}$ (4 marks)
- d) Let H be the set of all vectors of the form $\begin{bmatrix} c - a \\ a - 6b \\ 2a + b \end{bmatrix}$. Show that H is a subspace (3 marks)
- e) Define an operator $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(x, y, z) = (2x, -y, z)$. Determine whether T is linear. (5 Marks)
- f) Determine whether vectors $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$ are linearly independent if $\vec{V}_1 = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \vec{V}_2 = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$ and $\vec{V}_3 = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$ (4 marks)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Let $\vec{v}_1, \vec{v}_2 \in V$. Show that $H = \text{span}\{\vec{v}_1, \vec{v}_2\}$ is a subspace of V (6 marks)
- b) Determine whether the matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ is a zero of the polynomial $P(x) = x^3 + 2x^2 - 4x + 3$ (8 marks)
- c) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $f(t) = 2t - 3$. Find $f(A)$ (3 marks)
- d) Determine which properties of a vector space are not satisfied by $\mathbb{N}$ - the set of natural numbers. (3 marks)

QUESTION THREE (20 MARKS)

- a) Define a linear map T on $\mathbb{R}^3$ by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find:
- A basis and the dimension of the image U of T
 - A basis and the dimension of the kernel W of T
- (10 Marks)

b) Find the characteristic polynomial, eigenvalues and minimal polynomial of the matrix

$$A = \begin{pmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix} \quad (10 \text{ marks})$$

QUESTION FOUR (20 MARKS)

a) Let $T: M_2\mathbb{R} \rightarrow \mathbb{R}_2[x]$ be defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a+2b+3c+4d)x^2 + (2a-b+c)x + (3a-2b+c+2d)$. Find the matrix representation $[T]_E^F$ relative to the bases $E = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ and $F = \{x^2, x, 1\}$ for $M_2\mathbb{R}$ and $\mathbb{R}_2[x]$. (5marks)

b) Let V be the vector space of functions with basis $S = \{\sin t, \cos t, e^{3t}\}$ and let $D: V \rightarrow V$ be the differential operator defined by $D(f(t)) = \frac{d}{dt}(f(t))$. (4 marks).

c) Consider the following bases of $\mathbb{R}^2$.

$$S = \{e_1 = (1, -2), e_2 = (3, -4)\} \text{ and } S^* = \{f_1 = (1, 3), f_2 = (3, 8)\}$$

- Find the co-ordinates of $V = (a, b)$ relative to the basis S . (2 marks)
- Find the change of basis matrix P from S to S^* . (2 marks)
- Find the change of basis matrix Q from S^* to S . (1 mark)
- Verify that $Q = P^{-1}$. (1 mark)
- Show that $P^{-1}[v] = [v]_{S^*}$ for any vector $V = (a, b)$ in $\mathbb{R}^2$. (1 mark)

d) Suppose that B is similar to A , i.e. $B = P^{-1}AP$. Prove that $B^n = P^{-1}A^nP$ and so B^n is similar to A^n . (2 marks)

QUESTION FIVE (20 MARKS)

- a) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find:
- The determinant of A
 - $\text{Adj}(A)$

iii. A^{-1}

(10 Marks)

b)

i. State Cayley – Hamilton theorem.

ii. Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 5 & 3 \\ 2 & 10 \end{pmatrix}$

iii. Verify Cayley – Hamilton theorem for A

(10 Marks)