



GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS JANUARY-APRIL 2025 SEMESTER

BACHELOR OF EDUCATION(ARTS)

COURSE CODE: BSCS 210

COURSE TITLE: LINEAR ALGEBRA 1

DATE: APRIL 2025

TIME:

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY**.
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: All exam rooms are under CCTV surveillance during the examination period.

SECTION A: COMPULSORY

Question One

a) Consider $A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 4 & 5 \\ 2 & 3 & 6 \end{pmatrix}$; Evaluate,

i. $|A^2|$. (3 marks)

ii. A^{-1} (5 marks)

b) Determine whether $\{(1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5)\}$ is a basis of $\mathbb{R}^4$. (3 marks)

c) Find the distance between the parallel planes $2x - y + z = 1$ and $2x - y + z = -1$. (3mks)

d) Given that $A = \begin{pmatrix} 2 & 4 \\ -23 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 4 \\ 7 & -15 \end{pmatrix}$, find $4A + 2B^T$. (3 marks)

e) Express $P = t^2 + 4t - 3$ as a linear combination of the polynomials, $p_1 = t^2 - 2t + 5$, $p_2 = 2t^2 - 3t$, $p_3 = t + 3$. (5 marks)

f) Determine whether or not the following vectors are orthogonal.
 $\vec{a} = (\sin\theta, -\cos\theta)$, $\vec{b} = (\cos\theta, \sin\theta)$. (3 marks)

g) Solve the following system using Gauss-Jordan elimination method. (5 marks)

$$x + y + z = 9$$

$$2x + 4y - 3z = 1$$

$$3x + 6y - 5z = 0$$

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

- a) Are the following vectors linearly independent?
- i. $(1, -4, -3, 1), (1, -1, -1, 2), (3, -8, -2, 6).$ (3 marks)
 - ii. $\vec{a} = t^2 - 4t + 2, \vec{b} = t^2 + 2t + 4, \vec{c} = 2t^2 - t - 3.$ (3 marks)
- b) If A $(-2, 1)$, B $(2, 3)$ and C $(-2, -4)$ are three points, find the angle between lines AB and BC. (5 marks)
- c) Define a spanning set, hence/ otherwise, show that the set $\{(1, 1, 0), (-1, 1, 0), (1, 2, 1)\}$ spans $\mathbb{R}^3$. (5 marks)
- d) Write $(1, -3, 2)$ as a linear combination of $(2, -1, 1)$ and $(1, -8, 5)$. (4 marks)

QUESTION THREE (20 MARKS)

- a) Solve the following system by Gaussian elimination method. (5 marks)

$$2x + y + 2z = 10$$

$$x + 2y + z = 8$$

$$3x + y - z = 2$$

- b) Solve the following system using the Cramer's Rule method. (5 marks)

$$x_1 + 3x_2 + 2x_3 = 3$$

$$2x_1 + 4x_2 + 2x_3 = 8$$

$$x_1 + 2x_2 - x_3 = 10$$

- c) Find the parametric equations and symmetric equations of a line passing through the points $(2, -3, 4)$ and $(3, 5, -6)$. (5 marks)
- d) Find the reduced row echelon form of the following matrix. (5 marks)

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 5 \end{pmatrix}$$

QUESTION FOUR (20 MARKS)

- a) Let W be the subspace of $\mathbb{R}^4$ spanned by vectors $(1, 2, 0, -2), (2, 6, -3, -3), (3, 10, -6, -5)$. Find a basis and dimension of W . (5 marks)
- b) Find the parametric and the symmetric equations of the line which passes through the point $(4, -1, 9)$ and is perpendicular to the plane $3x - 2y + z = 18$. (3 marks)
- c) Determine whether the following vectors are linearly dependent or linearly independent. $(1, -3, 2), (2, -4, -1), (1, -5, 7)$. (3 marks)
- b. Show that the line $y = 2x + 3$ is a subspace of $\mathbb{R}^2$ (4 marks)
- c. Find the inverse of the below matrix by row echelon reduction form. (5 marks)

$$\begin{pmatrix} 2 & 1 & -1 \\ 5 & 2 & -3 \\ 0 & 2 & 1 \end{pmatrix}$$

QUESTION FIVE (20 MARKS)

- a) Let $\vec{a} = (1, -3, 2)$ and $\vec{b} = (2, -1, 1)$. Find;
- i. A scalar β such that $\vec{a}$ is orthogonal to $\vec{a} - \beta\vec{b}$. (3 marks)
 - ii. A unit vector that is orthogonal to both $\vec{a}$ and $\vec{b}$. (3 marks)
- b) Determine whether or not the following are subspaces of $\mathbb{R}^3$.
- i. $U = \{(x, y, z) \in \mathbb{R}^3 \mid x = 2y\}$. (4marks)

ii. $V = \{(a, b, c) \in \mathbb{R}^3 \mid ab = 0\}.$

(3marks)

c) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$

Find the inverse of the matrix A; hence use it to solve the following system of linear equations using the inverse method.

$$x + y + 2z = 0$$

$$x - y + z = 4$$

$$2x + y + z = 2$$

(7 marks)