GRETSA UNIVERSITY - THIKA

UNIVERSITY EXAMINATIONS

JANUARY-APRIL 2025

BACHELOR OF EDUCATION (ARTS)

COURSE CODE: BSCS 411

COURSE TITLE: REAL ANALYSIS

DATE: APRIL 2025 **TIME:**

INSTRUCTIONS TO CANDIDATES

1. SECTION A IS **COMPULSORY.**
2. SECTION B: ANSWER ANY OTHER **TWO** QUESTIONS.
3. **DO NOT** WRITE ANYTHING ON THIS QUESTION PAPER AS IT WILL BE AN EXAM IRREGULARITY.
4. ALL ROUGH WORK SHOULD BE AT THE BACK OF YOUR ANSWER BOOKLET AND CROSSED OUT.

CAUTION: *All exam rooms are under CCTV surveillance during the examination period.*

SECTION A: COMPULSORY

Question One

a) (i) Let S be a subset of $\mathbb{R}$. Give the definition of an accumulation point of the set S in $\mathbb{R}$. (2 marks)

(ii) Let S = $\{(-1)^n + \frac{1}{n}$; n∈ N$\}$. Find the accumulation point of the set S in $\mathbb{R}$. (1 mark)

(iii) When is a set S in $\mathbb{R}$ said to be closed ?. (1 mark)

(iv) Let u_n= [-1, $\frac{-1}{n}$] , n∈ N be a collection of closed sets. Using this collection, show clearly than an infinite union of closed sets in $\mathbb{R}$ is not necessarily closed. (3 marks)

b) Use definition of limit of a sequence to prove that $lim_{n\to\infty} \frac{2n^2+1}{n^2+1} = 2$. (4 marks)

c) Find the closure of the set A=$\{ (-1)^n\frac{n}{n+1}\}$ (3 marks)

d) Test for absolute or conditional convergence of the series $\sum_1^\infty \frac{(-1)^{n-1}}{n^2+1}$.

(4 marks)

e) Prove that $\sqrt{3}$ is irrational (4 marks)

f) The intersection of any finite collection of open sets is open in $\mathbb{R}$. Prove.

(5 marks)

g) Given that $A = \{2,3\}$ and $B = \{x, y, z\}$, find $A \times B$ and $A \times A$ (2 marks)

h) State the completeness axiom of real numbers. (1 mark)

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MARKS)

a) Show that $A = \{\frac{1}{2^n} : n \in \mathbb{N}\}$ is Cauchy (5 marks)

b) Suppose that $\{x_n\}\ and\ \{y_n\}$ are convergent sequences with $x_n \rightarrow$x and $y_n \rightarrow y\ as\ n \rightarrow \infty$. Show that $lim_{n\rightarrow\infty}$ (x_n + y_n) = x +y (5 marks)

c) Show that the sequence (x_n) in $\mathbb{R}$ has a unique limit. (5 marks)

d) Show that the function f(x) = $\frac{1}{x}$ is not uniformly continuous on (0, 1).

(5 marks)

QUESTION THREE (20 MARKS)

a) Show that the function f(x) = $\begin{cases} \frac{2x^2-3x+1}{x-1}; & x \neq 1; \\ -15 & x = 1; \end{cases}$

Is discontinuous at x=1. Hence/ otherwise, redefine f(x) so that it is continuous at x=1. (6 marks)

b) Show that finite sets have no limit points (4 marks)

c) Is the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^5}}$ conditionally convergent? (hint; use alternating series and p- series test) (6 marks)

d) Show that $\sum_{n=1}^{\infty} \frac{3}{10^n}$ converges. (4 marks)

QUESTION FOUR (20 MARKS)

a) Use the algebraic properties of all real numbers $\mathbb{R}$ to show that x.0 = 0 for all x$\in \mathbb{R}$. (3 marks)

b) Use Mathematical induction to prove that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 for all natural numbers n. (5 marks)

c) Find the supremum and infimum (if they exist) of each of the following sets. Again, decide which sets have maximum and minimum elements respectively.

S = {x$\in\mathbb{R}$; $x^2 < 5$ and x is a rational}. (2 marks)

T = $\{\frac{1}{n} + (-1)^2; n \; is \; a \; antural \; number\}$. (2 marks)

d) Suppose that r is a rational number and s is an irrational number such that t = r +s. Prove that t is irrational. (4 marks)

e) Prove that every convergent sequence is bounded. (4 marks)

QUESTION FIVE (20 MARKS)

a) Show that $w = \sqrt{n+1} - \sqrt{n-1}$ is irrational. (5 marks)

b) Prove that every convergent sequence is Cauchy. (4 marks)

c) Determine whether $f(x) = x^2 + 2x + 6$ is continuous at $x = 3$. (3 marks)

d) Let $a_n = \begin{cases} \frac{n}{2^n}; & n \; is \; odd \\ \frac{1}{2}, & n \; is \; even \end{cases}$. Does the series $\sum a_n$ converge?. (2 marks)

e) Which of the following series converge, which one diverge

i) $\sum \frac{n^2}{2^n}$. (3 marks)

ii) $\sum n!\, e^{-n}$ (3 marks)